Letter Grid (Spoiler)

An efficient way to count the words in the grid is based on dynamic programming. For each prefix of the word and for each cell of the grid, we calculate $D[p][y][x]$: the number of ways the first p letters of the word can be read from the grid so that the last letter is in the cell (y, x).

If $p = 1$, it's easy to calculate the value $D[p][y][x]$: if the cell (y, x) contains the first letter of the word, then $D[p][y][x] = 1$, and otherwise $D[p][y][x] = 0$. If $p > 1$ and the cell (y, x) contains the p'th letter of the word, we sum all the ways a one letter shorter prefix can end in a neighbour cell of (y, x). That is, $D[p][y][x] = \sum D[p-1][y'][x']$ where (y', x') is a neighbour cell of (y, x). Again, if the cell (y, x) doesn't contain the p'th letter of the word, $D[p][y][x] = 0$.

Let's calculate how many times the word TARTU appears in the example grid:

E	R	A	T
A	T	S	R
A	U	T	U

First $p = 1$ and the prefix is T:

0	0	0	1
0	1	0	0
0	0	1	0

Then $p = 2$ and the prefix is TA:

0	0	2	0
1	0	0	0
1	0	0	0

Then $p = 3$ and the prefix is TAR:

0	3	0	0
0	0	0	2
0	0	0	0

Then $p = 4$ and the prefix is TART:

0	0	0	2
0	3	0	0
0	0	2	0

Then $p = 5$ and the prefix is TARTU:

0	0	0	0
0	0	0	0
0	5	0	2

Finally, we count $5 + 2$ and obtain the desired answer 7.