Matching Bins (Spoiler)

Let

$$s_1, \ldots, s_n$$

be the input sequence of bins and $s_i \leq m$ for $i = 1, \ldots n$.
The k leftmost bins can be put into the next k bins in some order if and only if $2k \leq n$ and there exists a permutation p of the numbers $1, \ldots, k$, such that

$$s_i < s_{k+p(i)} \text{ for } i = 1, \ldots, k \tag{1}$$

The solution to the problem is the largest k satisfying condition (1).
Assume that k is a solution and p is a permutation satisfying condition (1). Let
$x = \max(s_1, \ldots, s_k)$ and $x = s_i$
$y = \max(s_{k+1}, \ldots, s_{k+k})$ and $y = s_{k+j}$
It is obvious that $x < y$. Assume that

$$p(i) = k + jj \tag{2}$$
$$p(ii) = k + j \tag{3}$$

Then

$$s_i < s_{k+jj} \leq s_{k+j}$$

$$s_{ii} \leq s_i < s_{k+jj}$$

Therefore we can modify the permutation p such that

$$p(i) = j, p(ii) = jj$$

and the modified p also satisfies condition (1).
Let $L(x)$ is the number of bins of size x in the first k bins, and $R(x)$ is the number of bins of size x in the next k bins, that is

$$L(x) = |\{i : 1 \leq i \leq k,\, s_i = x\}| \;\; x = 1, \ldots, m \tag{4}$$
$$R(x) = |\{j : k+1 \leq j \leq 2k,\, s_j = x\}| \;\; x = 1, \ldots, m \tag{5}$$

Define $R(m+1) = 0$ and $D(m+1) = 0$ and

$$D(x) = D(x+1) + R(x+1) - L(x) \;\; x = m, \ldots, 1 \tag{6}$$

Now we can state that the k leftmost bins can be put into the next k bins in some order if and only if condition (7) holds.

$$D(x) \geq 0 \;\; x = m, \ldots, 1 \tag{7}$$

Values of $L(x)$ and $R(x)$ can be computed in $\Theta(n)$ running time and then checking for the condition (7) takes $O(m)$ time. Therefore for a given value of k we can check whether k is a solution in $O(m \cdot n)$ time. Moreover, if we already computed the values of L and R for a given k then for $k-1$ we have to modify $L(k)$, $R(k)$, $R(2k)$ and $R(2k-1)$ only.
As a consequence, we obtain an $O(m \cdot n)$ worst case running time algorithm. The required memory is $\Theta(m+n)$.
We can speed up a bit by recomputing only those $D(x)$ that might be changed by decreasing the value of k.